Birzeit University
Mathematics Department
Math. 234

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Second Exam **Second Semester 99/2000**

Student Name:________________________________**Number:**___________ **Section**____

Q1 :(40 points) Circle the most correct answer

(1) Which of the following set of vectors is linearly dependent.

- **(a)** $(1,1,1)^T, (2,3,1)^T$
- **(b)** $(1,0,1)^T, (2,3,5)^T, (5,0,0)^T$
- **(c)** $1, x-1, x-x^2, x^2-x-1, 2x-5$
- **(d)** $\sin x, \cos x, x$

(2) Which of the following is not a basis for the corresponding space

- **(a)** $x, , x-1, x^2-x; P_3$
- **(b)** $(1,0,1)^T, (2,3,5)^T, (6,4,3)^T, (3,2,5)^T; R^3$
- **(c)** $x-2, 5; P_2$
- **(d)** $(1,1)^T, (2,5)^T; R^2$

(3) The coordinate vector of x^2-2x+3 with respect to the basis $[x, x-1, x^2+x]$ is

- **(a)** $(-1,1,1)^T$
- **(b)** $(0,-3,1)^T$
- **(c)** $(-1,3,1)^T$
- **(d)** $(1,-1,1)^T$

(4) If A is an $n \times n$ singular matrix then

- **(a)** $Nullity(A) = n$.
- **(b)** $Rank(A) = n$
- **(c)** $Nullity(A) = 0$
- **(d)** None of the above

(5) Let $S = \{ax^2 + bx + a - b : a, b \in R\}$. Then a basis and the dimension of S are

- **(a)** basis: $x^2, x, 1$; dimension $= 3$
- **(b)** basis: $x^2 + 1, x + 1$; dimension $= 2$
- **(c)** basis: $x^2 + x + 1$; dimension $= 1$
- **(d)** basis, $x^2 + 1, x - 1$; dimension $= 2$

(6) The dimension of the column space of $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix}$ is

- **(a)** 1
- **(b)** 2
- **(c)** 3
- **(d)** 4

(7) If A is an $n \times m$ matrix then

- **(a)** $0 \leq Rank(A) \leq \max(n, m)$
- **(b)** $1 \leq Rank(A) \leq \min(n, m)$
- **(c)** $n \leq Rank(A) = m$
- **(d)** $0 \leq Rank(A) \leq \min(n, m)$

(8) Which of the following is not a linear transformation

- **(a)** $L : R^3 \to P_3$, defined by $L((a, b, c)^T)$=$ax^2 + bx + c$
- **(b)** $L : M_{2\times2} \to P_3$,defined by $L(\begin{pmatrix} a & b \\ c & d \end{pmatrix})$=$ax^2 + bx + 2$
- **(c)** $L : R^3 \to R^2$, defined by $L((a, b, c)^T)$=$(a - b + c, a - b)^T$
- **(d)** $L : R^3 \to P_3$, defined by $L((a, b, c)^T)$=$ax^2 + bx + b - c$

(9) If V and W are subspases of a vector space U, $dim(U) = 3, dim(V) = dim(W) = 2$, then

- **(a)** $V \cup W$ is a subspace of U
- **(b)** $V \cap W = \{0\}$
- **(c)** $V \cap W \neq \{0\}$
- **(d)** none

(10) If A is an $n \times n$ nonsingular matrix then

- **(a)** $N(A) = \{0\}$
- **(b)** the rows and columns of A are linearly independent
- **(c)** $Rank(A) = n$
- **(d)** all of the above

Q2 :(20 points)(a) Let $L : V \to W$ be a linear transformation from a vector space V into a vector space W. Prove that L is one-to-one iff $KerL = \{\mathbf{0}\}$.

Recall that $L : V \to W$ is one-to-one if for any $v_1 \neq v_2 \in V$,then $L(v_1) \neq L(v_2)$.

b) Let U, V be two subspaces of a vector space W.

Let $U + V = \{u + v, u \in U, v \in V\}$. Prove that $U + V$ is a subspace of W.

Q3 :(40 points) Let $L : M_{2\times 2} \rightarrow P_3$ be a transformation defined by

$$L(\begin{pmatrix} a & b \\ c & d \end{pmatrix}) = (a+b)x^2 + (b-c)x - (c+d)$$

(a) Find a matrix representation of L with respect to the bases
$E = [\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}]$, and $F = [x^2 - 2, x + 2, 1]$.

(b) Find a basis for KerL.

(c) Find a basis for RangeL.

(d) Compute $[L(\begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix})]_F$