BIRZEIT UNIVERSITY

Mathematics Department

Math 330

Make Up Exam **Summer04/05**

Student name: …………………………………. ID no.: ………… sec……..

Circle the most correct answer: (SHOW YOUR WORK)

(1) To derive $\frac{3}{8}$ Simpson's rule we use lagrange polynomial with

(a) 2 nodes
(b) 3 nodes
(c) 4 nodes
(d) 5 nodes
(e) None of the above

(2) When using Huen's method to estimate the solution of the initial value problem $y' = f(x, y), y(a) = y_0$ on the interval $[a, b]$, find k such that

$E(y(b), h) = kE(y(b), \frac{h}{4})$.

(a) $\frac{1}{4}$
(b) 4
(c) $\frac{1}{16}$
(d) 16
(e) None of the above

(3) The multiplicity of the root $x = 1$ **of the polynomial** $p(x) = x^{101} - x^{100} - x + 1$ **is**

(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above

(4) Which of the following is a cubic spline

(a) $f(x) = \begin{cases} 11 - 24x + 18x^2 - 4x^3 & for \quad 1 \le x \le 2 \\ -54 + 72x - 30x^2 + 4x^3 & for \quad 2 < x \end{cases}$

(b) $f(x) = \begin{cases} 13 - 31x + 23x^2 - 5x^3 & for \quad 1 \le x \le 2 \\ -35 + 51x - 22x^2 + 3x^3 & for \quad 2 < x \le 3 \end{cases}$

(c) $f(x) = \begin{cases} \frac{19}{2} - \frac{81x}{4} + 15x^2 - \frac{13x^3}{4} & for \quad 1 \le x \le 2, \\ \frac{-77}{2} + \frac{207x}{4} - 21x^2 + \frac{11x^3}{4} & for \quad 2 \le x \le 3. \end{cases}$

(d) Non of the above
(e) More than one true answer

(5) The RMS error for the linear approximation $y = 8.6 - 1.6x$ **to the data points** $(-1,9.9), (0,9), (1,7)$ **is**

(a) 0.40
(b) 0.29
(c) 0.50
(d) 0.25
(e) Non of the above

(6) When using Newton Method to find the root $P=1$ of the function $f(x)=x^3+x^2+x-3$ the order of convergence R and the asymptotic error constant A are

(a) $R=1, A=\frac{2}{3}$

(b) $R=2, A=\frac{2}{3}$

(c) $R=1, A=\frac{1}{2}$

(d) $R=2, A=\frac{1}{2}$

(e) None of the above

(7) When using the Euler's method to solve the initial value problem:

$$y' = xy \quad ; \; a \le x \le b$$

$$y(a) = y_0 \; ; \; x_n = a+nh$$

Then y_3 **is given by**

(a) $y_3 = y_o(1+hy_o)^3$

(b) $y_3 = y_o(1+hx_o)(1+hx_1)(1+hx_2)$

(c) $y_3 = y_o(1+hx_1)(1+hx_2)(1+hx_3)$

(d) $y_3 = y_1(1+hx_1)(1+hx_2)(1+hx_3)$

(e) None of the above

(8) Use Huen's method:

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n+h, y_n+hf(x_n, y_n))\right] \; ; y_0 = y(x_0)$$

With $h=0.2$ **to approximate** $y=(0.2)$ **given that** $y' = y - t^2 + 1; y(0) = 0.5$

(a) 1.168
(b) 0.834
(c) 1.152
(d) 0.826
(e) None of the above

(9) Using four-digit arithmetic and Gaussian elimination without pivoting, the computed approximate solution to the system

$$1.333x_1 + 5.281x_2 = 6.414$$
$$24.14x_1 - 1.210x_2 = 22.93$$

Is

(a) (1.000, 1.000)
(b) (1.001,0.9956)
(c) (0.9981,9626.)
(d) None of the above

(10) Given that

$$x = g_1(x, y) = \frac{4x - x^2 + y + 3}{4}$$
$$y = g_2(x, y) = \frac{3 - xy + 2y}{2}$$

and $(p_0, q_0) = (2.1, 1.4)$
Use Siedel iteration to find (p_1, q_1)

(a) (2.795, 0.9435)
(b) (1.43, 2.018775)
(c) (2.0975, 1.43)
(d) (2.0975, 1.43175)
(e) None of the above

(11) The 3-point Gauss quadrature formula:

$$\int_{-1}^{1} f(x)dx \approx \frac{5}{9}f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}f(0) + \frac{5}{9}f\left(\sqrt{\frac{3}{5}}\right)$$

Has an error of the form $kf^{(6)}(\xi), \xi \in (-1,1)$. **Use the fact that the formula is not exact for polynomials of degree 6 or more to determine the value of the constant** k.

(a) 1/135
(b) 1/15750
(c) 8/175
(d) 1/2520
(e) None of the above

(12) Use the following data and lagrange polynomial to estimate $f(2.5)$

x_i	$f(x_i)$
0.5	2
1	1
2	0.5

(a) 1
(b) 0
(c) -1
(d) 0.5
(e) None of the above

(13) If the Trapeziodal rule:

$$\int_A^B f(x)dx = \frac{h}{2}\left[f_0 + 2f_1 + \ldots + 2f_{n-1} + f_n\right] - \frac{(b-a)}{12}h^2 f''(\xi); \xi \in (a,b)$$

is used to approximate $\int_1^2 x\ln(x)dx,$ **what is the largest value of** h **below so that the approximation is accurate within** 10^{-4} **?**

(a) 0.1
(b) 0.01
(c) 0.001
(d) 0.0001
(e) None of the above

(14) If the *LU*-decomposition: $A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$**,** What is l_{32}**.**

(a) 2/3
(b) -3/7
(c) -4/3
(d) 7/3
(e) None of the above

(15) Find the least square fit of the form $y = \frac{1}{Ax+B}$ **, for the given table**

x	y
-1	0.5
0	$\frac{1}{4}$
1	$\frac{1}{3}$

(a) $A = 3, B = 0.5$
(b) $A = 0.5, B = 3$
(c) $A = \frac{-1}{12}, B = \frac{13}{36}$
(d) $A = \frac{13}{36}, B = \frac{-1}{12}$
(e) None of the above

(16) What is the first iterate $x^{(1)}$ **when Newton's method is applied to approximate the solution to the nonlinear system:**

$$-x_1(x_1+1)+2x_2 = 18$$

$$(x_1-1)^2+(x_2-6)^2 = 25$$

With starting vector $x^{(0)} = [0,0]^t$**.**

(a) $[192,-48]^t$
(b) $[12,-3]^t$
(c) $[-12,3]^t$
(d) $[-192,48]^t$
(e) None of the above

(17) For the nonlinear equation: $x - \tan(x) = 0$**, which fixed-point arrangement below will converge for all** $x_0 \in [4,5]$?

(a) $x = \tan(x)$
(b) $x = \tan(x+\pi)$
(c) $x = \pi + \tan^{-1}(x)$
(d) $x = \tan^{-1}(x)$
(e) None of the above

(18) What is the order of the error of the forward formula

$$f''(x_0) = \frac{f_2 - 2f_1 + f_0}{h^2} \text{ ?}$$

(a) $\boldsymbol{O}(h^2)$
(b) $\boldsymbol{O}(h^3)$
(c) $\boldsymbol{O}(h)$
(d) $\boldsymbol{O}(h^4)$
(e) None of the above

(19) What is the order of approximation when estimating the functions $f(x) = e^{-x2}$ **by** $g(x) = 1 - x^2 + \frac{x^4}{2}$

(a) $\boldsymbol{O}(x^4)$
(b) $\boldsymbol{O}(x^6)$
(c) $\boldsymbol{O}(x^5)$
(d) $\boldsymbol{O}(x^3)$
(e) None of the above

(20) The voltage $E = E(t)$ **in an electrical circuit obeys the equation** $E(t) = L\frac{dI}{dt} + RI(t)$**, where** R **is resistance and** L **is inductance, use** $L = 0.05$ ***and*** $R=2$ **and values of** $I(t)$ **in the table below to find** $E(1.2)$**,**

t	$I(t)$
1.1	7.2428
1.2	5.9908
1.3	4.5260

(a) 11.3024
(b) -11.3024
(c) 12.6608
(d) None of the above