Birzeit University
Math. Dept.
Math. 337

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Final Exam **First Semester 2003/2004**

Q1 Prove or disprove the following:

- **(a)** any subgroup of an abelian group G is normal in G
- **(b)** If a group G is cyclic then every subgroup of G is abelian
- **(c)** If H is normal in G and K is a subgroup of G then $H \cap K$ is a normal subgroup of G.
- **(d)** A_4 is normal in S_4
- **(e)** If $\Phi : G \to H$ is a group isomorphism and K is a normal subgroup of G then $\Phi(K) \lhd H$
- **(f)** Every group of prime order is cyclic

Q2 **(a)** Prove that any subgroup H of a group G with index 2 is normal but not conversely.

- **(b)** Let G be a group of order $pq, p\&q$ are primes. Show that if $Z(G) \neq \{e\}$ then G is abelian.

Q3 **(a)** Let G be a group such that $a = a^{-1}, \forall a \in G$. Show that G is abelian.

- **(b)** Let H be a subgroup of G. Show that xHx^{-1} is a subgroup of G
- **(c)** Let H be the only subgroup of G of order n. Show that $H \lhd G$

Q4 **(a)** Let a be of order n. Prove that $|a^k| = \frac{n}{gcd(n,k)}$

- **(b)** List all generators of $Z_2 \oplus Z_9$.
- **(c)** Show that there is no permutation α such that $\alpha(12)\alpha^{-1} = (123)$

Q5 (a) Let H be a normal subgroup of G and $|H| = 2$. Show that $H \subset Z(G)$

(b) Let H be a normal subgroup of G. Show that for every $g \in G, h \in H$, there exists $k \in H$ such that $gh = kg$.

Q6 (a) State and prove the First isomorphism theorem.

(b) Let $\phi : G \to H$ be a homomorphism. Show that ϕ is one-to-one iff $ker\phi = \{e\}$.

(c) Let H, K be normal subgroups of G. If $K \subset H$, show that $(G/K)/(H/K) \cong G/H$