First Hour-Exam **First Semester 203-2004**

Student Name:________________________________**Number**:____________ **Section**____

Q1 **Prove each of the following**:

(**a**) Any group G which consists of 4 elements only must be abelian

(**b**) Any cyclic group G is abelian

(**c**) Any subgroup of cyclic group is cyclic

(**d**) A group which has no nontrivial proper subgroup must be cyclic

(**e**) Let G be a group and let $a, b \in G$. Show that $|ab| = |ba|$

(**f**) Suppose that G is a group which has exactly one nontrivial proper subgroup. Show that G is cyclic and $|G| = p^2$ for some prime p.

(**g**) Let G be a group and let $a \in G$ with $|a| = n$. Let k be a divisor of n. Show that that $|a^k| = n/k$

(**h**) Let G be a group and let $a, b \in G$ with $ab \in Z(G)$. Show that $ab = ba$.