

## Implicit Differentiation

**Example 1.** Find  $\frac{dy}{dx}$  if  $y^2 = x$ .

**Example 2.** Find  $\frac{dy}{dx}$  if  $y^2 = x^2 + \sin(xy)$ .

### Implicit Differentiation takes 4 steps

Step 1. Differentiate both sides of the equation with respect to  $x$ , treating  $y$  as a differentiable function of  $x$

Step 2. Collect the terms with  $\frac{dy}{dx}$  on one side of the equation.

Step 3. Factor out  $\frac{dy}{dx}$ .

Step 4. Solve for  $\frac{dy}{dx}$ .

**Example 3 (Tangent and Normal lines).** Show that the point  $(2, 4)$  lies on the curve  $x^3 + y^3 - 9xy = 0$ . Then find the tangent and normal to the curve there.

**Example 4.** Find  $d^2y/dx^2$  if  $2x^3 - 3y^2 = 8$ .

**Theorem.** *If  $n$  is a rational number, then  $x^n$  is differentiable at every interior point of the domain of  $x^{n-1}$ , and*

$$\frac{d}{dx}x^n = nx^{n-1}$$

**Example 5.** (a)  $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

(b)  $\frac{d}{dx}(x^{\frac{2}{3}}) = \frac{2}{3}(x^{-\frac{1}{3}}) = \frac{2}{3x^{\frac{1}{3}}}$

(c) Find  $\frac{d}{dx}(1 - x^2)^{\frac{1}{4}}$

(d) Find  $\frac{d}{dx}(\cos x)^{-\frac{6}{5}}$