

The Mean Value Theorem

Theorem. [Rolle's Theorem] *Suppose that $y = f(x)$ is continuous at every point of $[a, b]$ and differentiable at every point of (a, b) . If*

$$f(a) = f(b) = 0,$$

then there is at least one number $c \in (a, b)$ at which $f'(c) = 0$

Theorem. [The mean value theorem] *Suppose that $y = f(x)$ is continuous on a closed interval $[a, b]$ and differentiable on the interval's interior (a, b) . Then there is at least one point $c \in (a, b)$ at which*

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Example 1. The function $f(x) = x^2$ is continuous for $0 \leq x \leq 2$, and differentiable for $0 < x < 2$. $f(0) = 0, f(2) = 4$. The mean value theorem says that, there exists $c \in (0, 2)$ such that $f'(x) = \frac{4-0}{2-0} = 2$

Some Consequences

Corollary. If $f'(x) = 0$ at each point of an interval I , then $f(x) = C$ for all $x \in I$, where C is a constant.

Corollary. If $f'(x) = g'(x)$ at each point of an interval I , then there exists a constant C such that $f(x) = g(x) + C$, for all $x \in I$.

Example 2. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$

Example 3. Find the velocity $v(t)$ and position $s(t)$ of a body falling freely from rest with acceleration 9.8 m/sec^2 .

Differential Equations

A differential equation is an equation relating an unknown function and one or more of its derivatives. **A solution** of a differential equation is a function whose derivatives satisfy the equation.

Example 4. 1) The function $s(t) = 4.9t^2$ is a solution to the equation $d^2s/dt^2 = 9.8$

2) The function $y = -\cos x + 3$ solves the equation $dy/dx = \sin x$

Example 5. A heavy projectile is fired straight up from a platform 3m above the ground, with an initial velocity 160 m/sec. Assume that the only force affecting the projectile during its flight is from gravity, which produces a downward acceleration of 9.8 m/sec^2 . Find an equation for the projectile's height above the ground as a function of t , if $t = 0$ when the projectile is fired. How high above the ground is the projectile 3 sec after firing.